



Propagación hacia adelante:

Para una muestra:

$$z_i = x_i w + b$$

$$a_i = \sigma(z) = (1) z_i$$

↑
Lineal

$$\hat{y}_i = a_i$$

última capa

Para un lote:

$$Z = X_{m \times 1} W_{1 \times 1} + B_{1 \times 1}$$

$$A = \sigma(Z) = IZ$$

↑
Lineal

$$\hat{Y} = A$$

última capa

Propagación hacia atrás:

Para una muestra:

$$\text{MSE: } L_i = (y_i - \hat{y}_i)^2$$

$$\text{MAE: } L_i = |y_i - \hat{y}_i|$$

$$J = L_i$$

Para un lote:

$$\text{MSE: } L = (y - \hat{y})^2$$

$$\text{MAE: } L = |y - \hat{y}|$$

$$J = \frac{1}{m} \sum_{i=1}^m L_i$$

Ejemplo:

	Year Center Norm (X)	Log N Transistors (Y)
0	0.2680	21.2539
1	0.9363	21.3651
2	1.0849	22.4280
3	-0.2518	16.4182

$$\alpha = 0.1$$

Epoca 1: $\begin{cases} w = -0.25 \\ b = 0 \end{cases}$

$$\hat{y} = 0.2680 \times -0.25 + 0 = -0.0670$$

$$J = L = [21.2539 - (-0.0670)]^2 = 434.58$$

↗ Costo

$$\frac{\partial J}{\partial b} = -2 [21.2539 - (-0.0670)] = -42.64$$

$$\frac{\partial J}{\partial w} = -2 [21.2539 - (-0.0670)] [0.2680] = -11.4280$$

Actualizar θ :

$$b := 0 - (0.1) (-42.64) = 4.264 \quad \uparrow \text{Incrementa}$$

$$w := -0.25 - (0.1) (-11.42) = 1.392 \quad \uparrow \text{Incrementa}$$

Epoca 2: $\begin{cases} w = 1.392 \\ b = 4.264 \end{cases}$

$$\hat{y} = 0.9363 \times 1.392 + 4.264 = 5.5673$$

$$J = L = [21.3651 - (5.5673)]^2 = 249.57$$

↗ Costo

$$\frac{\partial J}{\partial b} = -2 [21.3651 - (5.5673)] = -31.5956$$

$$\frac{\partial J}{\partial w} = -2 [21.3651 - (5.5673)] [0.9363] = -29.583$$

Actualizar θ :

$$b := 4.264 - (0.1) (-31.5956) = 7.4236 \quad \uparrow \text{Incrementa}$$

Gradiente:

$$\theta = [b, w]$$

$$\theta := \theta - \alpha \nabla_{\theta} J(\theta)$$

$$\nabla_{\theta} = \left[\frac{\partial}{\partial b}, \frac{\partial}{\partial w} \right]$$

$$\nabla_{\theta} J(\theta) = \left[\frac{\partial J}{\partial y} \frac{\partial y}{\partial b}, \frac{\partial J}{\partial y} \frac{\partial y}{\partial w} \right]$$

↗ Para un lote

$$\nabla_{\theta} J(\theta) = \left[\sum_{i=1}^m -2(y_i - \hat{y}_i), \sum_{i=1}^m -2(y_i - \hat{y}_i) x_i \right]$$

$$\nabla_{\theta} J(\theta) = \left[-2(y_1 - \hat{y}_1), -2(y_1 - \hat{y}_1) x_1 \right]$$

Atualizar θ :

$$b := 4.264 - (0.1) \cdot (-31.5956) = 7.4236 \quad \uparrow \text{Incrementar}$$

$$w := 1.392 - (0.1) \cdot (-29.583) = 3.45 \quad \uparrow \text{Incrementar}$$

$$\text{Epoca } 3: \begin{cases} w = 3.45 \\ b = 7.4236 \end{cases}$$

$$\hat{y} = 1.0899 \times 3.45 + 7.4236 = 11.1665$$

$$J = L = \left[22.4280 - (11.1665) \right]^2 = 126.8213$$

$\uparrow$ Custo

$$\frac{\partial J}{\partial b} = -2 \left[22.4280 - (11.1665) \right] = -22.523$$

$$\frac{\partial J}{\partial w} = -2 \left[22.4280 - (11.1665) \right] [1.0899] = -24.43$$

Atualizar θ :

$$b := 7.4236 - (0.1) \cdot (-22.523) = 9.6759 \quad \uparrow \text{Incrementar}$$

$$w := 3.45 - (0.1) \cdot (-24.43) = 5.893 \quad \uparrow \text{Incrementar}$$